

MESON-EXCHANGE CURRENTS AND THE
STRANGENESS RADIUS OF ${}^4\text{He}^*$

M. J. Musolf**

*Department of Physics
Old Dominion University
Norfolk, VA 23529 U.S.A.*

and

*CEBAF Theory Group, MS 12H
Newport News, VA 23606 U.S.A.*

and

T.W. Donnelly

*Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.*

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** N.S.F. Young Investigator

ABSTRACT

Meson-exchange current contributions to the strangeness radius of ${}^4\text{He}$ are computed in the one-boson exchange approximation. It is found that these contributions introduce a $\lesssim 10\%$ correction to the one-body contribution. They should not, therefore, hamper the extraction of the nucleon strangeness radius from the parity-violating electron- ${}^4\text{He}$ asymmetry.

There has been considerable interest recently in the use of intermediate-energy semileptonic scattering to study the strange-quark content of the nucleon [1-10]. The elastic neutrino-nucleon and anti-neutrino nucleon cross sections are particularly sensitive to the nucleon's strange-quark axial vector form factor [1,9,10]. The parity-violating (PV) elastic electron-proton and electron-nucleus asymmetries, on the other hand, can be sensitive to the nucleon's strange-quark vector current matrix elements [2-9]. The latter are parameterized by two form factors: the strangeness electric ($G_E^{(s)}$) form factor, which must vanish at $Q^2 = 0$ since the nucleon has no net strangeness, and the strangeness magnetic ($G_M^{(s)}$) form factor. Accordingly the leading Q^2 -dependence of the former may be characterized by a mean-square “strangeness radius” [11], which one may define as a dimensionless parameter ρ_s [7,9]:

$$\rho_s = \left. \frac{dG_E^{(s)}}{d\tau} \right|_{\tau=0} , \quad (1)$$

where $\tau = |Q^2|/4m_N^2$ (here, $Q^2 = q^2 - \omega^2 \leq 0$ is the square of the four-momentum transfer with $q = |\vec{q}|$ the three-momentum transfer and ω the energy transfer). Similarly, one defines a strange magnetic moment as $\mu_s = G_M^{(s)}(0)$.

Ideally, one would attempt to determine ρ_s and μ_s with a series of PV electron scattering experiments using a proton target. As discussed elsewhere, however, such a strategy would not necessarily permit a separation of these two parameters at the level of precision needed to distinguish among theoretical models [7-9, 11-16]. In principle, augmenting a PV $\vec{e}p$ scattering program with measurements of the PV asymmetry for scattering from a nucleus could permit a more precise determination of ρ_s and μ_s [7-9]. To that end, the $(J^\pi, T) = (0^+, 0)$ nuclei represent an attractive case. The PV asymmetry for elastic scattering from such targets depends on the ratio of isoscalar weak neutral current (NC) and isoscalar electromagnetic (EM) Coulomb matrix elements. To a large extent, the dependence of these matrix elements on details of the nuclear wave function cancels out from their ratio, making the asymmetry primarily sensitive to electroweak couplings and single-nucleon electric form factors. For purposes of extracting information on the nucleon's strangeness radius, one would like to estimate the scale of nuclear corrections to the $(0^+, 0)$ asymmetry. In this note, we investigate one class of nuclear corrections — meson-exchange currents (MEC) — to the strangeness radius of ^4He , a nucleus which will be used as a target in up-coming CEBAF experiments [17-19]. We find that, in the one-boson-exchange approximation, these corrections are sufficiently small so as not to

introduce serious uncertainty in a determination of ρ_s from the ${}^4\text{He}$ asymmetry. MEC contributions to the non-leading Q^2 -behavior of the ${}^4\text{He}$ strangeness form factor will be discussed in a forthcoming publication [20].

The PV asymmetry for scattering from a $(0^+, 0)$ nucleus is given by [7-9]

$$A_{LR} = -\frac{G_\mu |Q^2|}{4\sqrt{2}\pi\alpha} \left\{ \sqrt{3}\xi_V^{T=0} + \xi_V^{(0)} \frac{\langle \text{g.s.} \| \hat{M}_0(s) \| \text{g.s.} \rangle}{\langle \text{g.s.} \| \hat{M}_0(T=0) \| \text{g.s.} \rangle} \right\}, \quad (2)$$

where G_μ is the Fermi constant measured in muon decay, α the EM fine structure constant, and $\xi_V^{T=0}$ and $\xi_V^{(0)}$ isoscalar and SU(3)-singlet vector current couplings of the Z^0 to the nucleon, respectively [7-9]. The operators $\hat{M}_0(s)$ and $\hat{M}_0(T=0)$ are the Coulomb multipole projections of the strange-quark vector current ($\bar{s}\gamma_\mu s$) and isoscalar EM current, respectively, and $\langle \text{g.s.} \| \cdot \| \text{g.s.} \rangle$ denotes a ground state reduced matrix element. At tree level in the Standard Model, the electroweak couplings are $\sqrt{3}\xi_V^{T=0} = -4\sin^2\theta_w$ and $\xi_V^{(0)} = -1$. In the one-body approximation, the nuclear operators contained in the multipole projections are identical, apart from the single-nucleon form factors, so that the ratio of their matrix elements becomes

$$\frac{\langle \text{g.s.} \| \hat{M}_0(s) \| \text{g.s.} \rangle}{\langle \text{g.s.} \| \hat{M}_0(T=0) \| \text{g.s.} \rangle} \Big|_{1\text{-body}} = \frac{G_E^{(s)}(Q^2)}{G_E^{T=0}(Q^2)}, \quad (3)$$

independent of the details of the nuclear wave functions. At low momentum transfers, one has $G_E^{T=0} = 1/2 + \mathcal{O}(\tau)$ and $G_E^{(s)} = \rho_s \tau + \mathcal{O}(\tau^2)$, so that the scale of the second term in Eq. (2) is essentially determined by the nucleon's strangeness radius.

Meson-exchange current corrections to this result are generated by processes illustrated in the diagrams of Figs. 1 and 2. Inclusion of the currents in Fig. 1, where the exchanged meson is typically one of the lowest-lying pseudoscalar or vector mesons, is required in order to maintain consistency between the nucleon-nucleon interaction and conservation of EM charge and baryon number. For isoscalar vector currents, such as $J_\mu^{EM}(T=0)$ and $\bar{s}\gamma_\mu s$, only the “pair current” ($N\bar{N}$) processes of Fig. 1a,b contribute. The amplitude associated with the “meson-in-flight” process of Fig. 1c vanishes, since by G-parity one has that $\langle M | V_\mu(T=0) | M \rangle = 0$ for any meson M and isoscalar neutral current $V_\mu(T=0)$. When the exchanged meson is a pion and pseudovector coupling is used at the πNN vertices, the corresponding two-body isoscalar EM and strangeness Coulomb operators go like $q^2 F_1(T=0)$ and $q^2 F_1^{(s)} \sim q^4$, respectively, where F_1 is the Dirac form factor of the nucleon. Hence, the longest-range MEC's – those resulting from

single π -exchange – will not contribute to the nuclear strangeness radius. In the case of the ρ - and ω -meson exchanges, the corresponding two-body isoscalar Coulomb operators are also of $\mathcal{O}(q^2)$ times a linear combination of the electric and magnetic nucleon form factors. Thus, when analyzing the leading $|Q^2|$ -dependence of the ratio in Eq. (3), we may neglect these contributions to the EM matrix element appearing in the denominator, since they will only contribute to that ratio in $\mathcal{O}(q^4)$. In contrast, the nuclear strangeness radius (the numerator of Eq. (3)) receives a contribution from these currents, since $G_M^{(s)}$ does not necessarily vanish at the photon point. The resultant two-body operator has the form

$$\begin{aligned} \hat{M}_0^V(s) \Big|_{q \rightarrow 0} = & \tau \sum_{i < j} \left[\frac{g_V^2 m_V}{24\pi^{3/2} m_N} \right] \hat{T}_V(i, j) (1 + \kappa_V) \\ & \times \left[\frac{e^{-m_V r}}{m_V r} \right] (1 + m_V r) \left[1 + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j + \cdots \right] , \end{aligned} \quad (4)$$

where $V = \rho$ or ω ; $g_V \equiv g_{VNN}$ gives the vector meson-nucleon coupling; r is the relative separation of the two nucleons; the sum is performed over all distinct pairs of nucleons labelled (i, j) ; and

$$\hat{T}_V(i, j) = \begin{cases} \vec{\tau}_i \cdot \vec{\tau}_j, & V = \rho \\ 1, & V = \omega \end{cases} . \quad (5)$$

In writing Eq. (4) we have neglected terms $(+\cdots)$ which give non-vanishing matrix elements only if the nucleon pair carries orbital angular momentum $L > 0$ either internally or with respect to the nuclear center of mass. For the case of ${}^4\text{He}$, these terms only contribute to the Coulomb matrix elements via configuration mixing (*e.g.*, D-state admixtures into the ground state), and therefore may be neglected for the present purposes of setting a scale.

Isobar currents of the type shown in Fig. 2a contribute to the nuclear strangeness radius only when the intermediate nucleon resonance carries the same isospin as the nucleon, since the strange-quark vector current is a (strong) isospin singlet operator. The lightest $T = 1/2$ resonance is the $N(1440)$. Although we have not computed the contribution of the corresponding isobar current explicitly, we neglect it on the assumption that it is suppressed by the large mass splitting between this resonance and the nucleon.

A third class of MEC contributions – the so-called “transition currents” illustrated in Fig. 2b – involves the matrix element of $\bar{s}\gamma_\mu s$ between two different mesons, M and M' .

The lightest such pair is the ρ and π , whose strange vector current matrix element may be written as

$$\langle \rho^a(k_1, \varepsilon) | \bar{s} \gamma_\mu s | \pi^b(k_2) \rangle = \frac{g_{\rho\pi}^{(s)}(Q^2)}{m_\rho} \epsilon_{\mu\nu\alpha\beta} k_1^\nu k_2^\alpha \varepsilon^{\beta*} \delta_{ab} \quad , \quad (6)$$

where ε^β is the ρ -meson polarization and (a, b) are isospin indices. Note that there is no analogous $\omega - \pi$ matrix element since the strange vector current is isoscalar. In contrast to the situation with diagonal $\bar{s} \gamma_\mu s$ currents, no conservation principle requires the form factor $g_{\rho\pi}^{(s)}(Q^2)$ to vanish at the photon point. Consequently, the *nuclear* Coulomb matrix element of the transition charge operator derived from diagram 2b can, in principle, contribute to the nuclear strangeness radius. Unlike the ρ -meson pair-current contribution embodied in Eq. (4), the $\rho - \pi$ transition contribution bears no connection with the nuclear potential used in computing nuclear wave functions. It thus generates an unambiguous many-body correction to the one-body form of the PV asymmetry. The corresponding two-body Coulomb operator is, to leading order in q^2 ,

$$\begin{aligned} \hat{M}_0^{\rho\pi}(s) \Big|_{q \rightarrow 0} &= -\tau \sum_{i < j} \left(\frac{2}{9} \right) \left[\frac{g_{\pi NN} g_{\rho NN} g_{\rho\pi}^{(s)}}{\pi^{3/2}} \right] \vec{\tau}_i \cdot \vec{\tau}_j \\ &\times (1 + \kappa_\rho) \frac{1}{m_\rho r} \left(\frac{1}{m_\rho^2 - m_\pi^2} \right) [m_\pi^2 e^{-m_\pi r} - m_\rho^2 e^{-m_\rho r}] \vec{\sigma}_i \cdot \vec{\sigma}_j + \dots \quad , \end{aligned} \quad (7)$$

where terms with vanishing matrix elements in a ground state of pure S-waves have been omitted. In obtaining the two-body operators in Eqs. (4) and (7), we did not include form factors and the hadronic vertices. For purposes of obtaining an upper bound on the scale of MEC corrections, however, the use of point meson-nucleon couplings should be sufficient. For comparison, the one-body strangeness Coulomb operator has the low- q^2 form

$$\hat{M}_0^{(1)}(s) \Big|_{q \rightarrow 0} = \frac{\tau}{2\sqrt{\pi}} \rho_s \sum_i^A 1 \quad . \quad (8)$$

To set the scale of the different one- and two-body contributions to the strangeness radius of ^4He , we computed matrix elements of the operators in Eqs. (4,7,8) using a simple ^4He ground state wave function consisting of a Slater determinant of harmonic oscillator single-particle S-state wave functions. Since the one-body operator in Eq. (8) does not probe the nuclear wave function but simply counts the number of nucleons, its matrix element is not dependent on our choice of wave function. The two-body matrix

elements, in contrast, are more sensitive to the choice of wave function. A particularly important consideration in this respect is the role played by short-range nucleon-nucleon anti-correlations, since all of the MEC's treated here involve at least one heavy vector meson propagator. The pieces of the resultant two-body operators associated with the heavy meson will have an effective range $\sim 1/m_V \lesssim 0.25$ fm for $m_V \gtrsim m_\rho$. Consequently, their matrix elements will carry a non-negligible sensitivity to the short-distance part of the nuclear wave function. In principle, one could account for this sensitivity by using Monte Carlo methods and variational ground state wave functions to evaluate the two-body matrix elements [21]. For purposes of setting the scale of the MEC contribution, however, it is sufficient to use the simpler harmonic oscillator wave function and to account for short-range anti-correlations by including a phenomenological correlation function, $g(r)$, in the integral over relative co-ordinate, r . Following Ref. [22], we take this function to have the form

$$g(r) = C \left[1 - e^{-r^2/d^2} \right] \quad , \quad (9)$$

where C is a constant adjusted to maintain the wave function normalization and the parameter $d = 0.84$ fm is obtained by fitting the nuclear matter correlation function of Ref. [23]. With this choice for $g(r)$, the two-body matrix elements can be evaluated analytically, thereby making transparent the physical parameters which govern the scale of these matrix elements.

Looking first at the limit of un-correlated wave functions ($g(r) \equiv 1$), we obtain for the sum of one- and two-body contributions to the strange-quark Coulomb matrix element

$$\langle \text{g.s.} \parallel \hat{M}_0(s) \parallel \text{g.s.} \rangle \Big|_{q \rightarrow 0} = \tau \left[\lambda_1 \rho_s + \lambda_{2a} \mu_s + \lambda_{2b} g_{\rho\pi}^{(s)} \right] \quad , \quad (10)$$

where

$$\lambda_1 = 2/\sqrt{\pi} \quad (11a)$$

$$\lambda_{2a} \approx - \sum_{V=\rho,\omega} (1 + \kappa_V) \left[\frac{\sqrt{2}g_V^2}{8\pi^2} \right] \left[1 - \frac{5}{(m_V b)^2} + \dots \right] \left(\frac{m_V}{m_N} \right) \frac{\mathcal{N}_V}{(m_V b)^3} \quad (11b)$$

$$\lambda_{2b} = \frac{2\sqrt{2}}{9\pi^2} g_{NN\rho} g_{\pi NN} (1 + \kappa_\rho) \frac{\mathcal{N}_2}{m_\rho b} \left[\frac{1}{(m_\rho b)^2 - (m_\pi b)^2} \right] \quad (11c)$$

$$\times \left[(m_\pi b)^2 I(m_\pi b) - (m_\rho b)^2 I(m_\rho b) \right] \quad ,$$

where

$$I(mb) = 1 - \sqrt{\frac{\pi}{2}} (mb) \exp\left(\frac{(mb)^2}{2}\right) \operatorname{erfc}\left(\frac{mb}{\sqrt{2}}\right) , \quad (12)$$

and where b is the oscillator parameter, $\mathcal{N}_{V,2}$ are spin-isospin matrix elements, and g_V is the vector meson-nucleon coupling. From a fit to the ^4He charge form factor, one obtains a value for the oscillator parameter of $b = 1.2$ fm [9]. In this case, one has $m_\rho b \gg 1$ and

$$(m_\rho b)^2 I(m_\rho b) \approx 1 - \frac{3}{(m_\rho b)^2} + \dots . \quad (13)$$

Numerically, the one-body matrix element gives $\lambda_1 \approx 1.13$, while use of the un-correlated wave function gives $\lambda_{2a}^{(\rho)} \approx -0.06$ and $\lambda_{2a}^{(\omega)} \approx -0.03$ for the ρ - and ω -meson pair currents, respectively, and $\lambda_{2b} \approx -0.9$ for the transition current. In obtaining these results, we have used values for the couplings taken from Ref. [21]: $g_{NN\rho} = 2.6$, $g_{\omega NN} = 14.6$, $\kappa_\rho = 6.6$, and $\kappa_\omega = -0.12$. Although the scale of the un-correlated two-body matrix elements is suppressed with respect to the one-body matrix element by several powers of $1/(m_V b)$, this suppression is compensated in the case of the transition current by the large values of the meson-nucleon couplings and spin-isospin matrix element, \mathcal{N}_2 . Thus, in a world where anti-correlations became important only for $N - N$ separations $\ll 1/m_V$, the ^4He strangeness radius could be as sensitive to non-nucleonic strangeness as to the polarization of the nucleon's strange sea ($|\lambda_{2b}| \sim |\lambda_1|$).

In the actual world, the ^4He strangeness radius is dominated by the one-body (single nucleon) contribution. Using the correlation function of Eq. (9) in computing the Coulomb matrix elements, we obtain the same value of λ_1 as before and the following values for the two-body contributions: $\lambda_{2a}^{(\rho)} \approx -0.03$, $\lambda_{2a}^{(\omega)} \approx -0.015$, and $\lambda_{2b} \approx -0.06$. Whereas the pair-current contributions are reduced by a factor two from their un-correlated values, the transition current term is an order of magnitude smaller. The latter, more significant suppression arises because the transition current matrix element involves the difference of two operators, whose ranges are set, respectively, by $1/m_\pi$ and $1/m_\rho$ (see Eq. (7)). Short-range correlations reduce the value of the second operator's matrix element but do not seriously affect the first, so that the degree of cancellation between the two is enhanced. This cancellation is somewhat sensitive to the values of b and d employed, with λ_{2b} ranging from $\approx -0.1 \rightarrow -0.01$ as d is increased from $\sim 0.8 \rightarrow 1.0$ fm for $b = 1.2$ fm. In contrast, the values of $\lambda_{2a}^{(\rho, \omega)}$ are stable with respect to this variation.

In terms of the PV ^4He asymmetry, it is the ratio of the matrix element in Eq. (10) to the isoscalar EM Coulomb matrix element which governs the second term in Eq. (2). Since

the numerator of this term is already of $\mathcal{O}(\tau)$, we need retain only the $\tau = 0$ part of the denominator when extracting the nuclear strangeness radius from a low- $|Q^2|$ measurement of the asymmetry. Consequently, we set the denominator $= G_E^{T=0}(0)\lambda_1 = \lambda_1/2$. The resulting ratio is

$$\frac{\langle \text{g.s.} \| \hat{M}_0(s) \| \text{g.s.} \rangle}{\langle \text{g.s.} \| \hat{M}_0(T=0) \| \text{g.s.} \rangle} \Big|_{q \rightarrow 0} = 2\tau\rho_s \left[1 + \left(\frac{\lambda_{2a}}{\lambda_1} \right) \mu_s + \left(\frac{\lambda_{2b}}{\lambda_1} \right) g_{\rho\pi}^{(s)} \right] + \mathcal{O}(\tau^2) \quad . \quad (14)$$

From these results we conclude that the one-boson-exchange MEC's should not seriously impact the extraction of ρ_s from the helium asymmetry, unless μ_s and $g_{\rho\pi}^{(s)}$ are an order of magnitude larger in scale than ρ_s . The latter possibility seems unlikely, since a variety of calculations give $|\mu_s| \lesssim |\rho_s|$ [11-16]. Similarly, an estimate of $g_{\rho\pi}^{(s)}$ using a ϕ -dominance model and the measured width for $\phi \rightarrow \rho\pi$ gives $|g_{\rho\pi}^{(s)}| \approx 0.26$ [24], which is of the same magnitude or smaller than most estimates for the magnitude of ρ_s .

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FIGURE CAPTIONS

Fig. 1. “Pair-current” (a,b) and “meson-in-flight” (c) meson-exchange current (MEC) processes contributing to nuclear matrix elements of the electromagnetic (EM) and vector strange-quark currents. Here, N and N' denote two nucleons, M a meson, and the crossed circle either the EM or vector strange-quark current operators. The process of Fig. 1c does not contribute to the nuclear strangeness radius or isoscalar EM charge form factor.

Fig. 2. Isobar (a) and transition current (b) MEC processes contributing the the nuclear strangeness radius. The notation is identical to that of Fig. 1, with N^* denoting a nucleon resonance and (M, M') any two mesons.